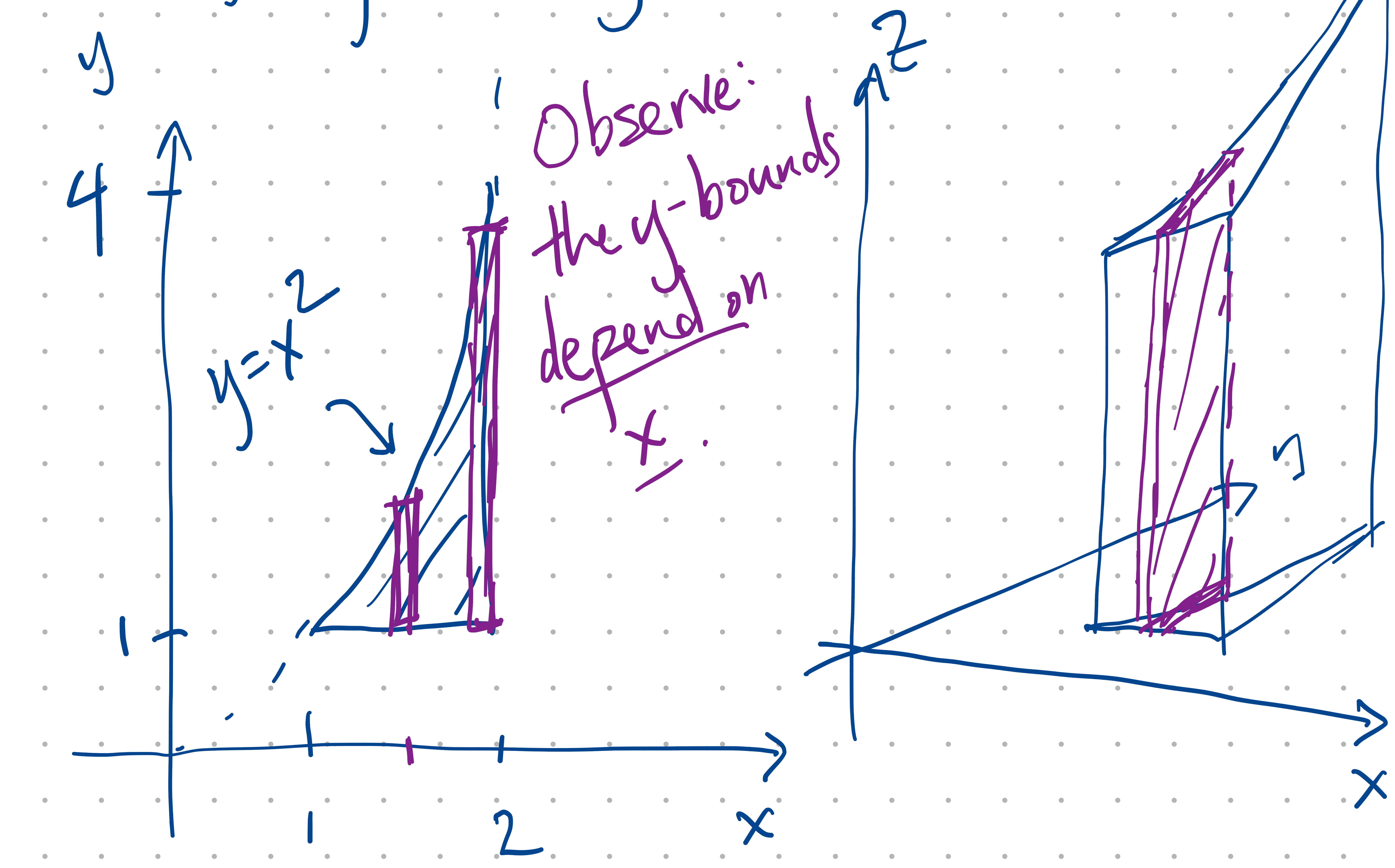
§15.2 OVERVIEW

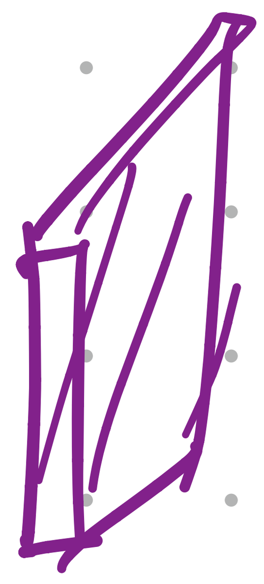
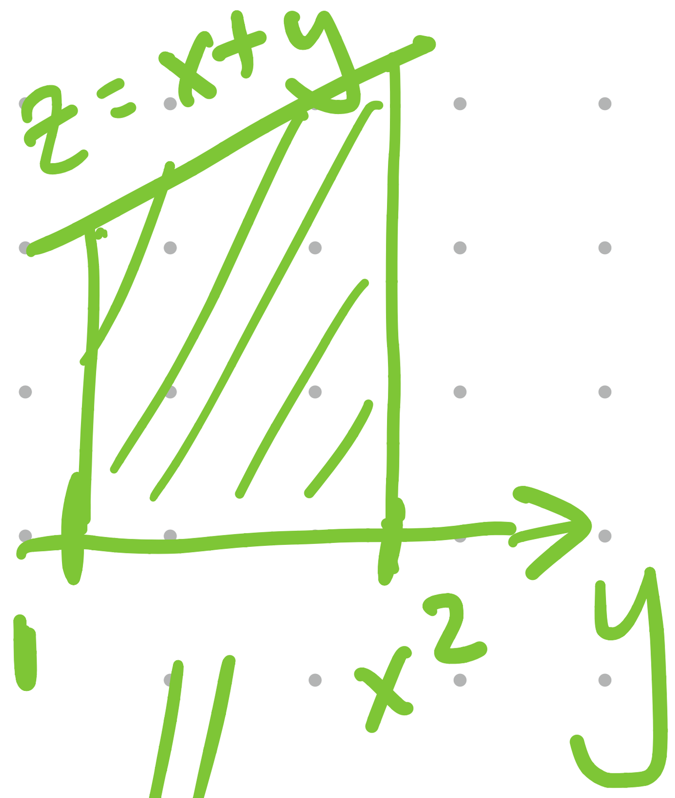
Previously, examined situation of integrating $f(x,y)$ over some rectangular region in the xy-plane

Today let's try to drop that restriction.

Ex $f(x,y) = x+y$



Same overall idea: cut into slices

Volume of slice  = "() dx"

$$= \int_1^{x^2} (x+y)\,dy$$

Total volume:

$$\int_1^2 \left(\int_1^{x^2} (x+y)\,dy \right) dx$$

So if integrating dydx, the y-bounds are allowed to depend on x, but the x-bounds (outer bounds) must be constants.

Let's compute:

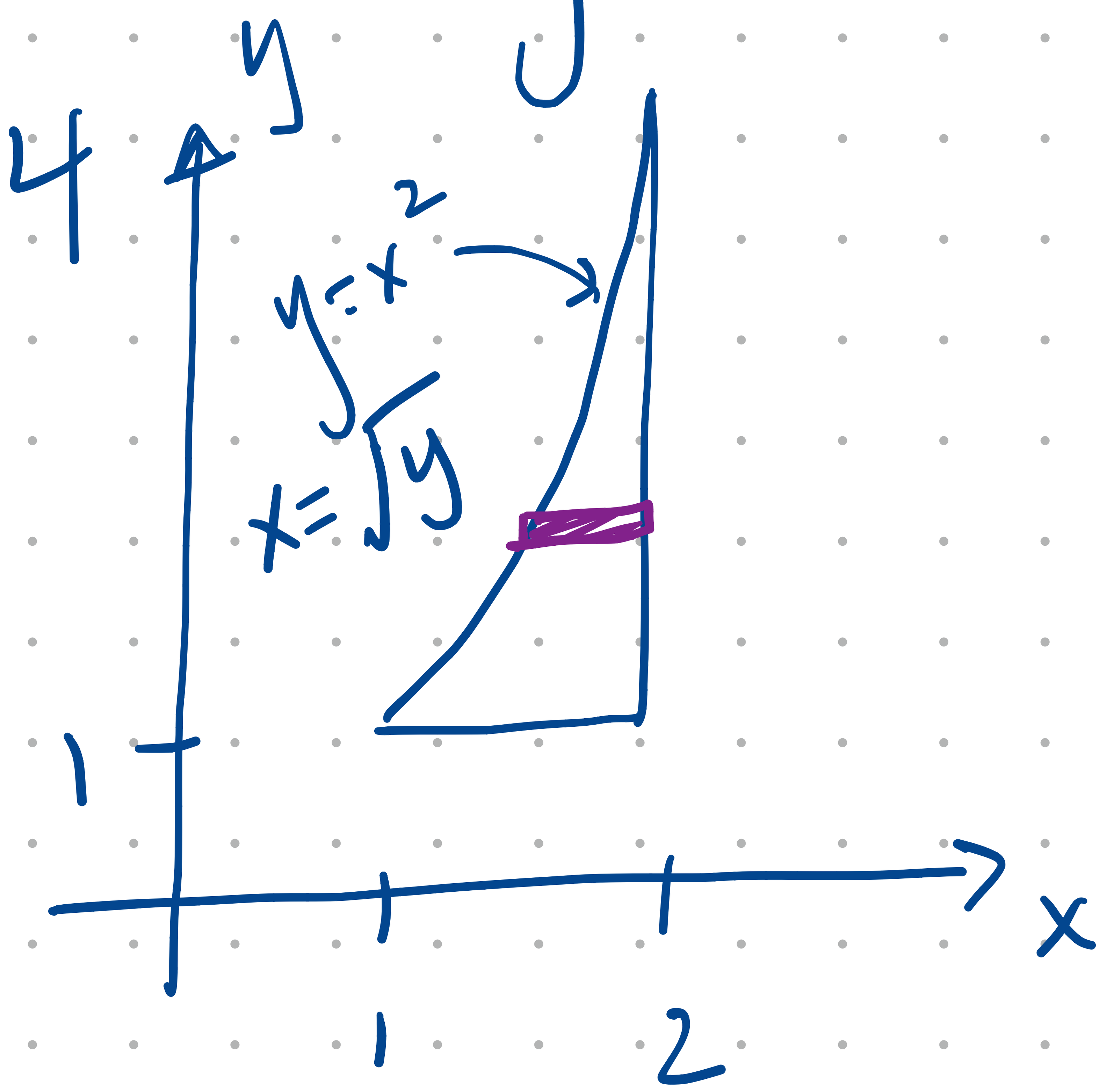
$$\int_1^2 \int_1^{x^2} (x+y)\,dy\,dx = \int_1^2 \left(xy+\frac{1}{2}y^2\right)\Bigg|_{y=1}^{y=x^2} dx$$

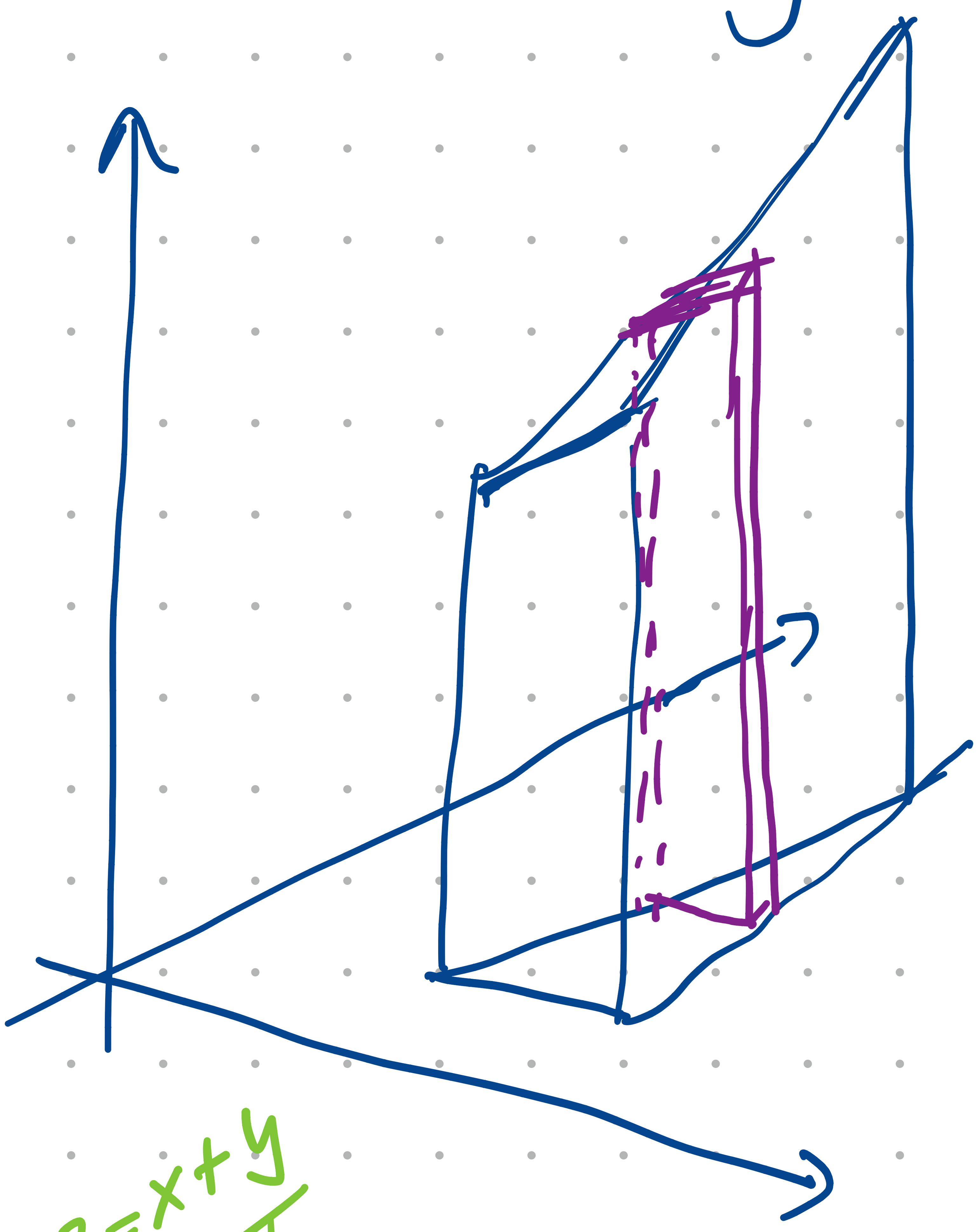
$$= \int_1^2 \left(x^3+\frac{1}{2}x^4\right)-\left(x+\frac{1}{2}\right)dx$$

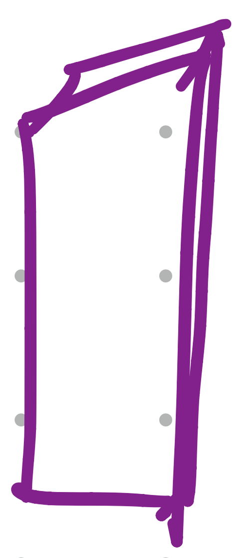
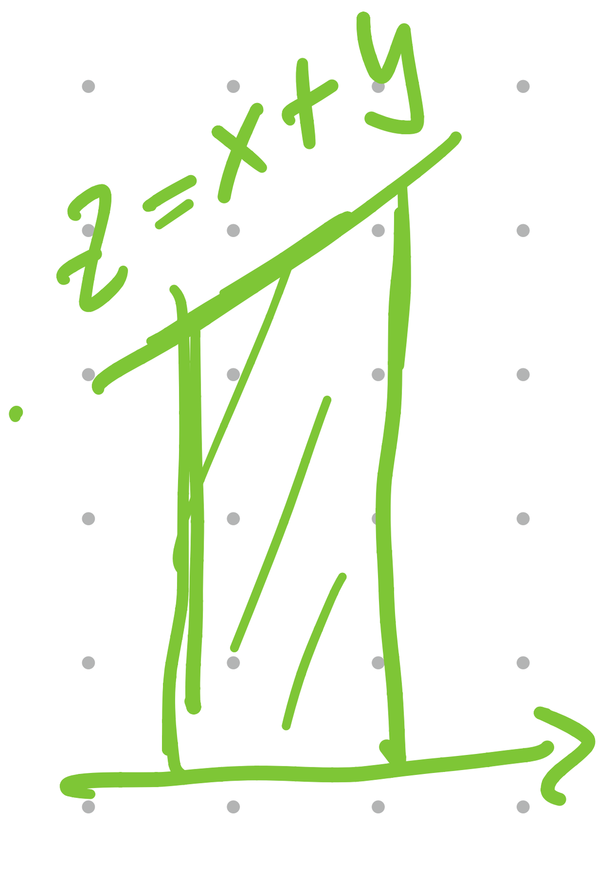
$$= \left(\frac{1}{4}x^4+\frac{1}{10}x^5-\frac{1}{2}x^2-\frac{1}{2}x\right)\Bigg|_{x=1}^{2}$$

$$= 97/20$$

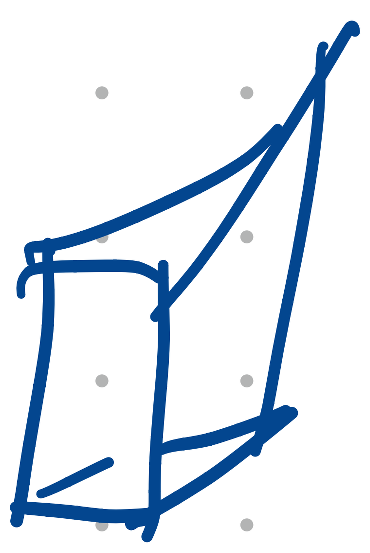
Just like last time, we can slice the region another way:



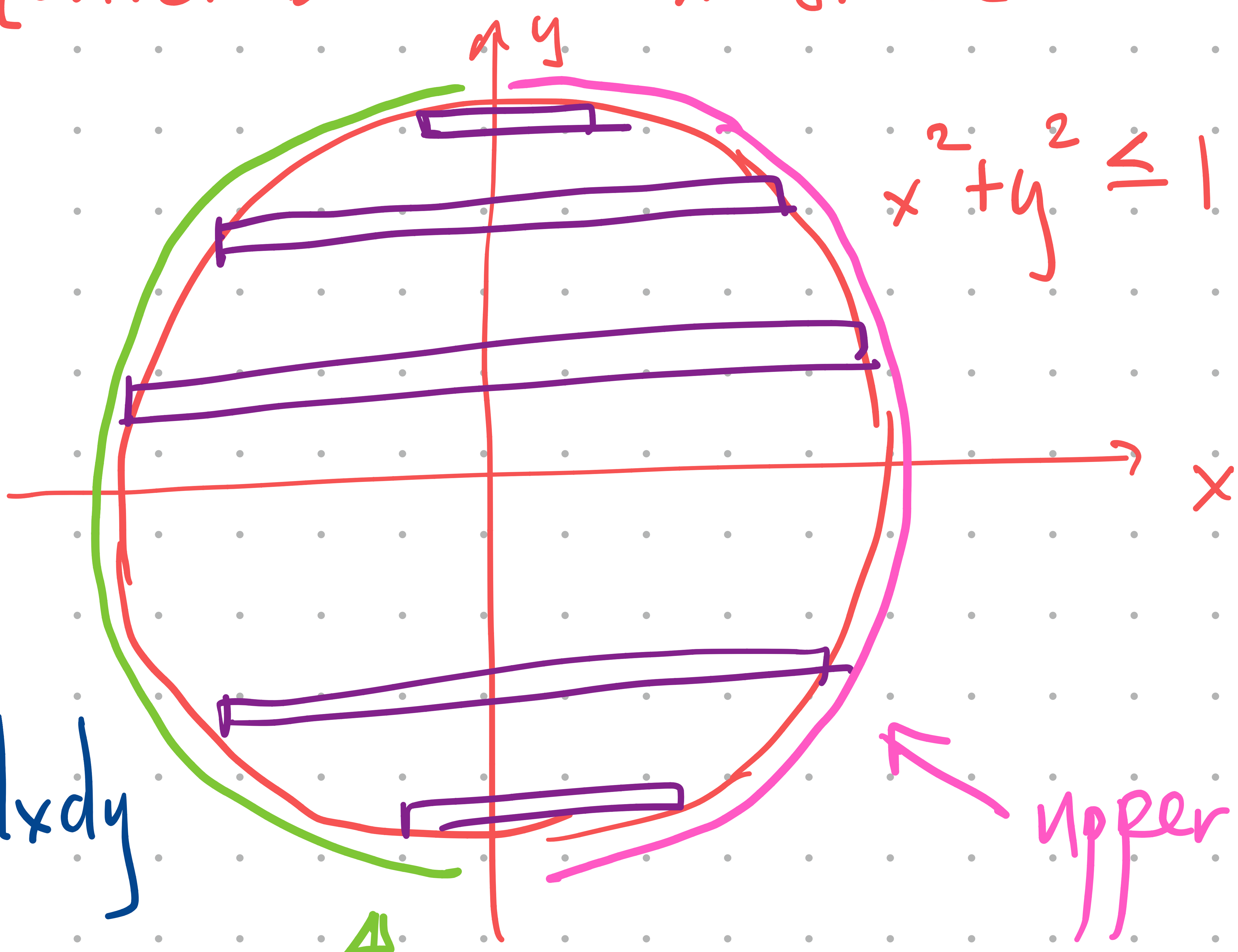


Volume of  = "() dy"

$$= \int_{\sqrt{y}}^{2} (x+y)\,dx$$

Volume of  $= \int_1^4 \left(\int_{\sqrt{y}}^{2} (x+y)\,dx \right) dy$

When doing $dx\,dy$ integration order, x bounds are allowed to depend on y, but y bounds (outer bounds) must be constants.



$$\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\,dx\,dy$$

upper x-bound

$x = \sqrt{1-y^2}$

lower x bound

$x = -\sqrt{1-y^2}$

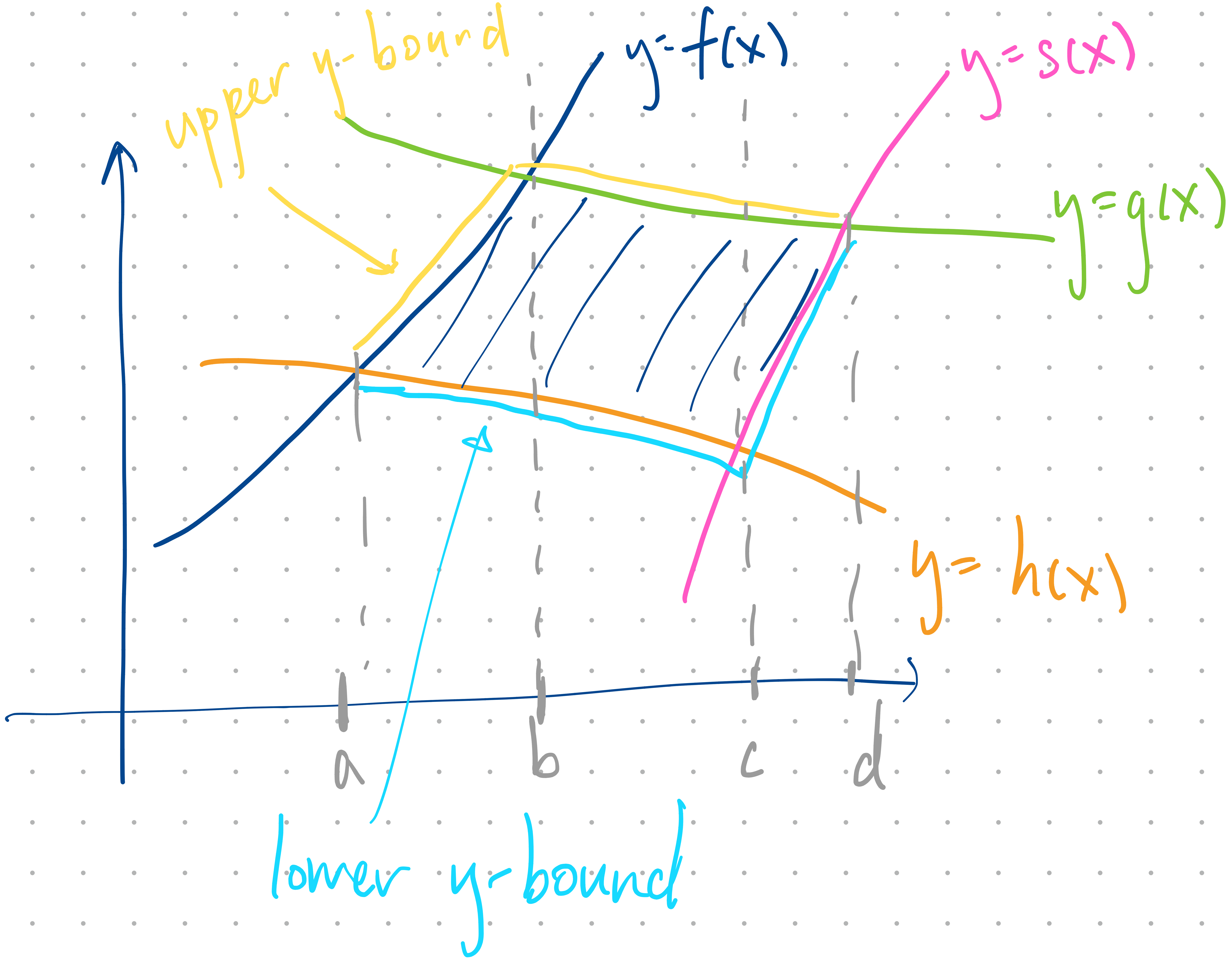
Compute:

$$\int_1^4 \int_{\sqrt{y}}^2 (x+y)\,dx\,dy = \int_1^4 \left(\frac{1}{2}x^2 + xy\right)\Big|_{x=\sqrt{y}}^2 dy$$

$$= \int_1^4 (2+2y) - \left(\frac{y}{2} + y^{3/2}\right) dy$$

$$= \left(2y + \frac{3}{4}y^2 - \frac{2}{5}y^{5/2}\right)\Big|_{y=1}^4$$

$$= 97/20$$



If we wanted to integrate $u(x,y)$ over this region:

$$\int_a^b \int_{h(x)}^{f(x)} u(x,y)\,dy\,dx + \int_b^c \int_{h(x)}^{g(x)} u(x,y)\,dy\,dx + \int_c^d \int_{s(x)}^{g(x)} u(x,y)\,dy\,dx$$

When poll is active, respond at **PollEv.com/xianglongni346**

Text **XIANGLONGNI346** to **37607** once to join

Suppose we want to integrate $f(x, y)$ over the parallelogram bounded by the lines $y = x, y = 2x, y = x + 1, y = 2x - 1$. (Draw a picture.) If we use the integration order $dydx$, how many integrals do we have to write? (Try to actually write them.)

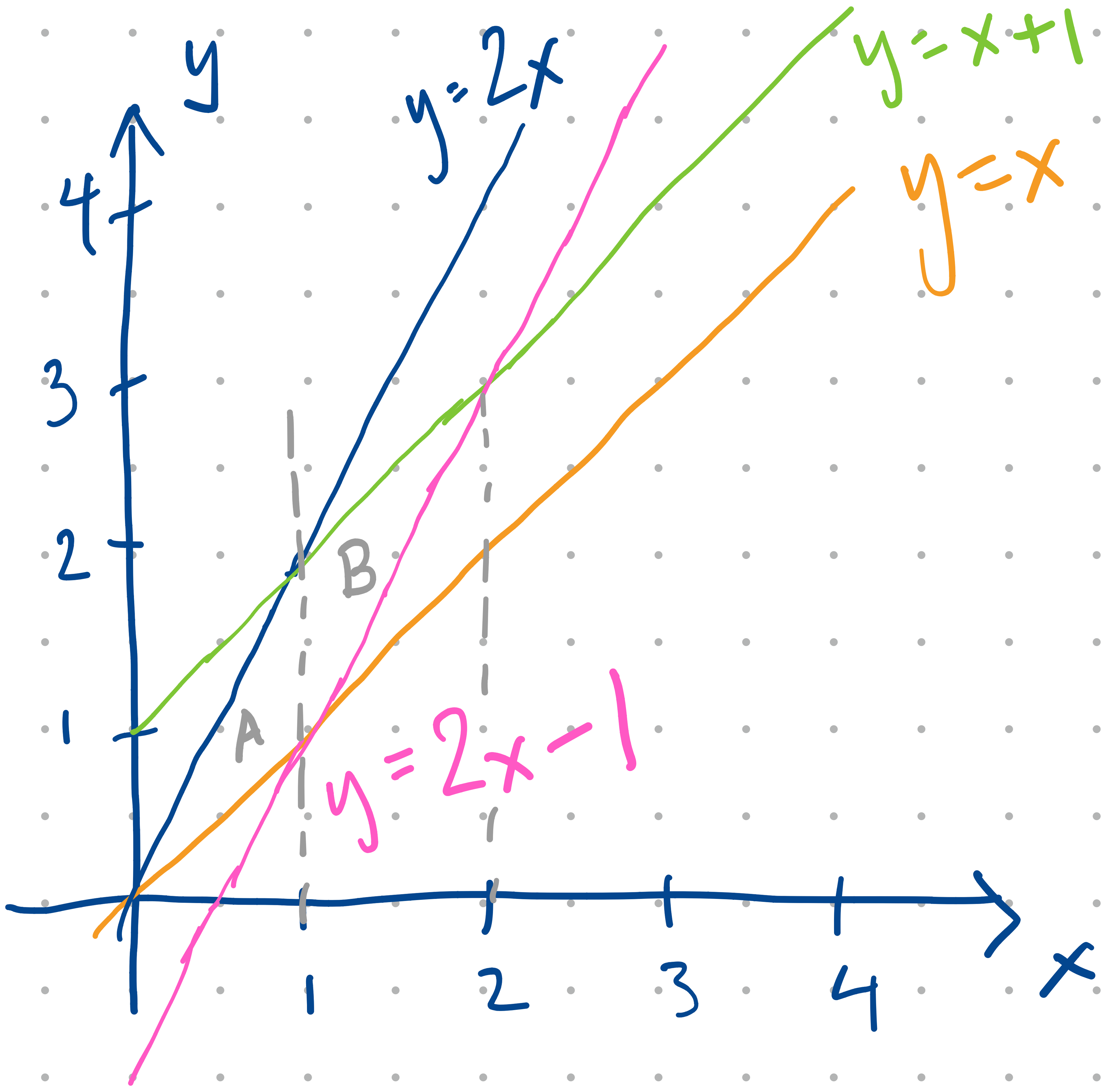
- 1
- 2
- 3
- More than 3

Total Results: 9

Suppose we want to integrate $f(x, y)$ over the parallelogram bounded by the lines $y = x, y = 2x, y = x + 1, y = 2x - 1$. (Draw a picture.) If we use the integration order $dxdy$, how many integrals do we have to write? (Try to actually write them.)

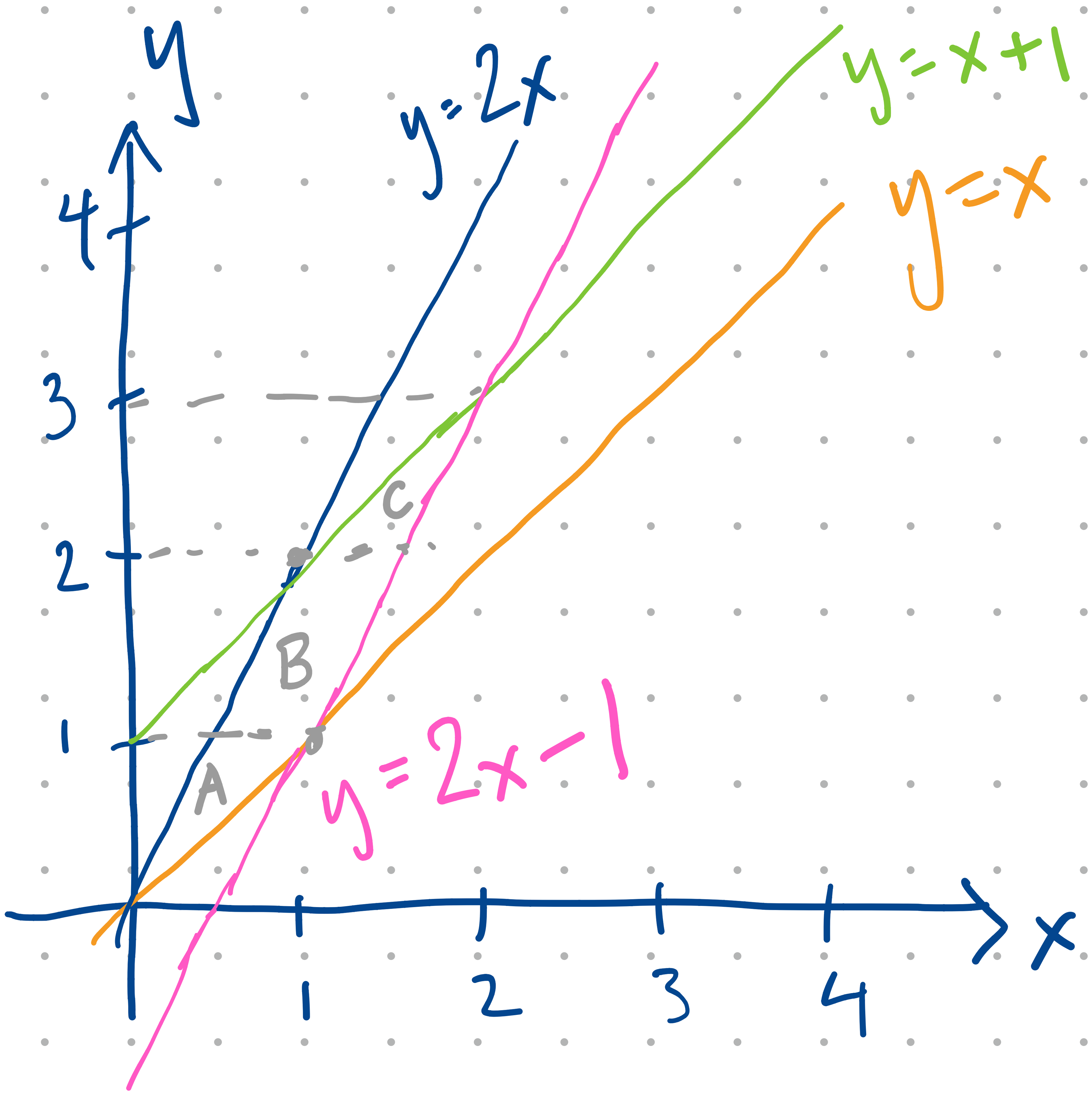
- 1
- 2
- 3
- More than 3

Total Results: 8



$$\int_0^1 \int_x^{2x} f(x,y)\,dy\,dx \quad A$$

$$+\int_1^2 \int_{2x-1}^{x+1} f(x,y)\,dy\,dx \quad B$$



$$\int_0^1 \int_{y/2}^{y} f(x,y)\,dx\,dy \quad A$$

$$+\int_1^2 \int_{y/2}^{\frac{y+1}{2}} f(x,y)\,dx\,dy \quad B$$

$$+\int_2^3 \int_{y-1}^{\frac{y+1}{2}} f(x,y)\,dx\,dy \quad C$$