

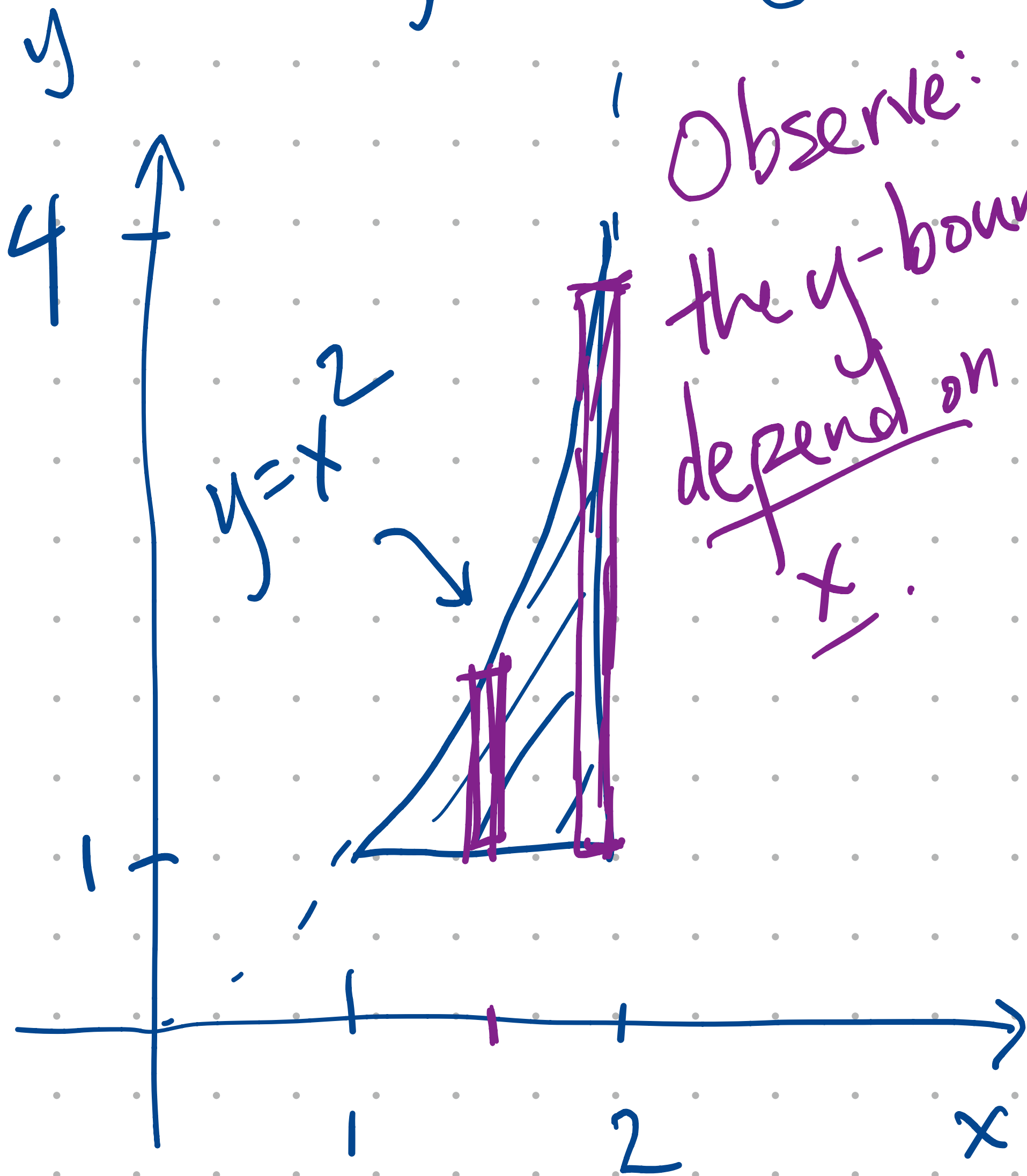
§15.2 OVERVIEW

Previously, examined situation of integrating

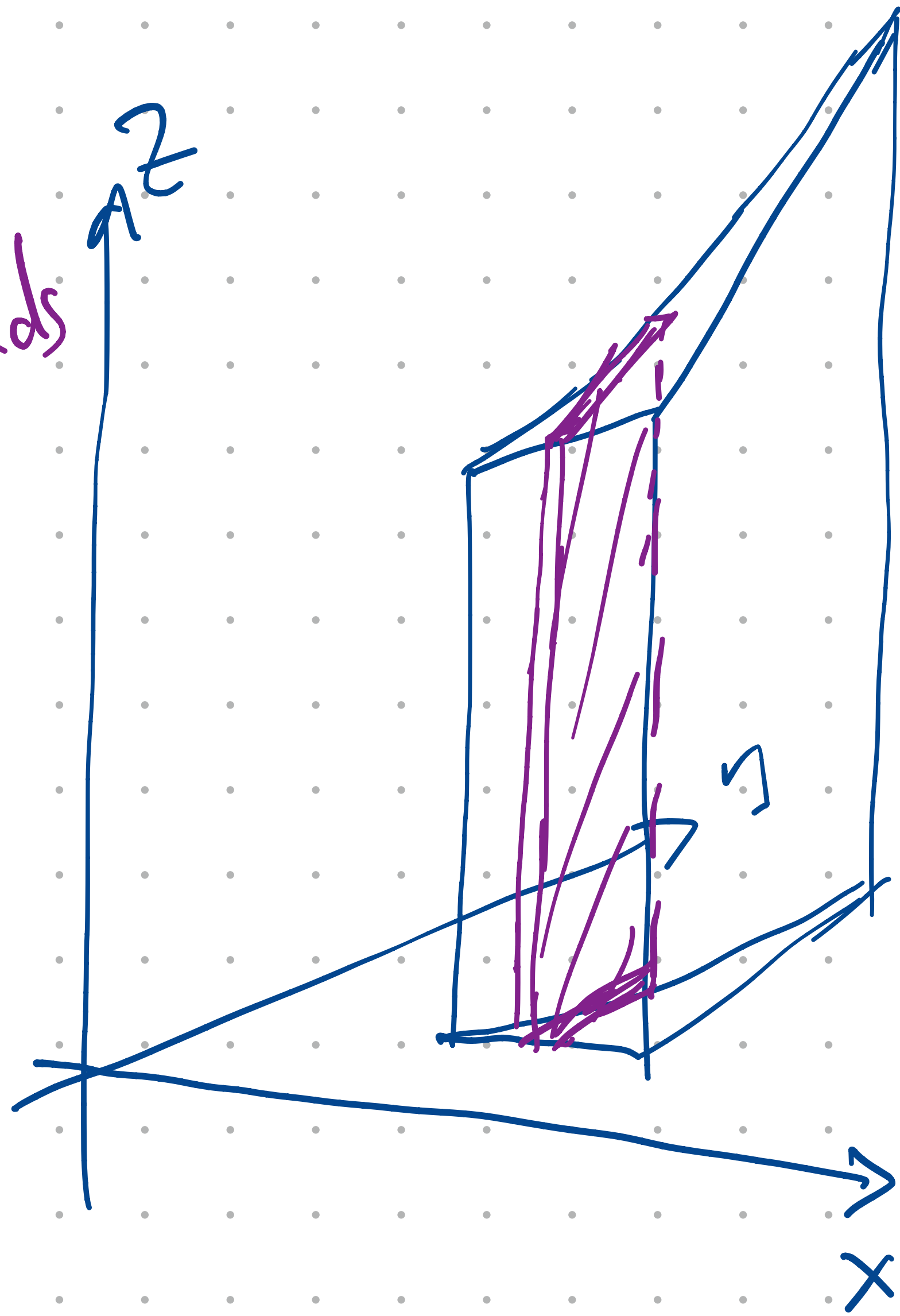
$f(x,y)$ over some rectangular
region in the xy -plane

Today let's try to drop that restriction.

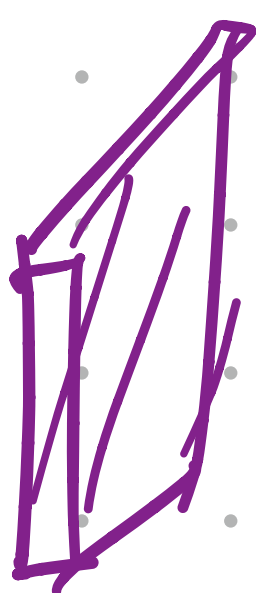
Ex $f(x,y) = x+y$

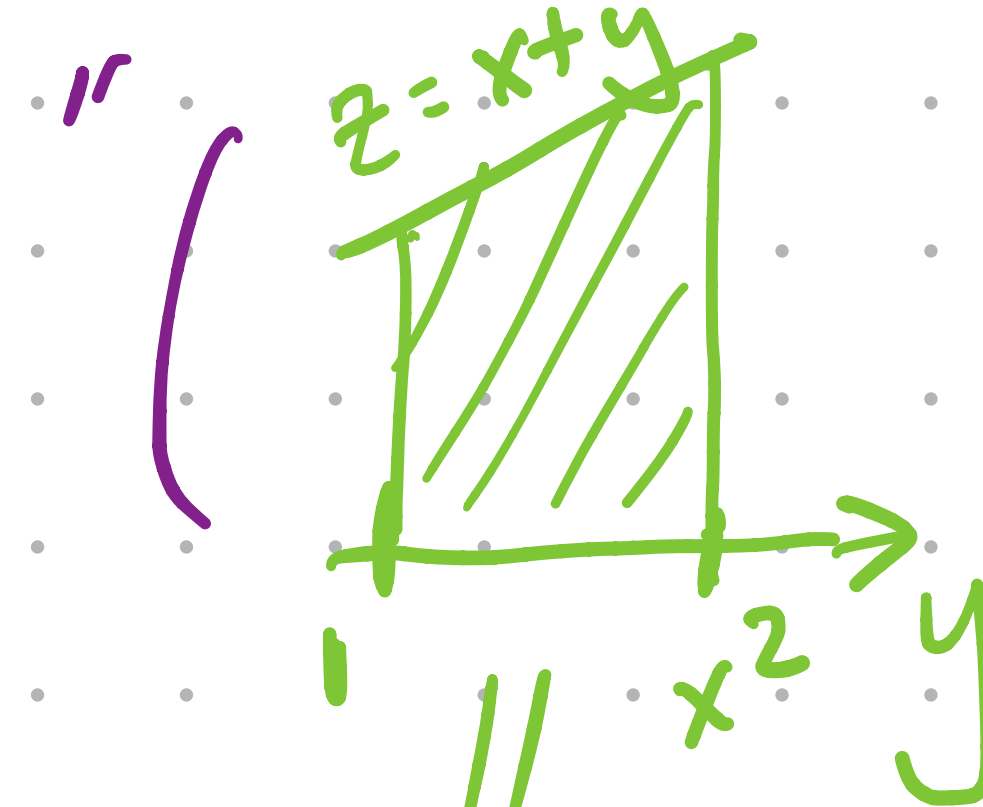


Observe:
the y -bounds
depend on
 x .



Same overall idea: cut into slices

Volume of slice  = $\left(\int_{-x^2}^{x^2} (x+y) dy \right) dx$



$\int_{-x^2}^{x^2} (x+y) dy$

Total volume:

$$\int_1^2 \left(\int_{-x^2}^{x^2} (x+y) dy \right) dx$$

So if integrating $dy dx$, the y -bounds are allowed to depend on x , but the x -bounds (outer bounds) must be constants.

Let's compute:

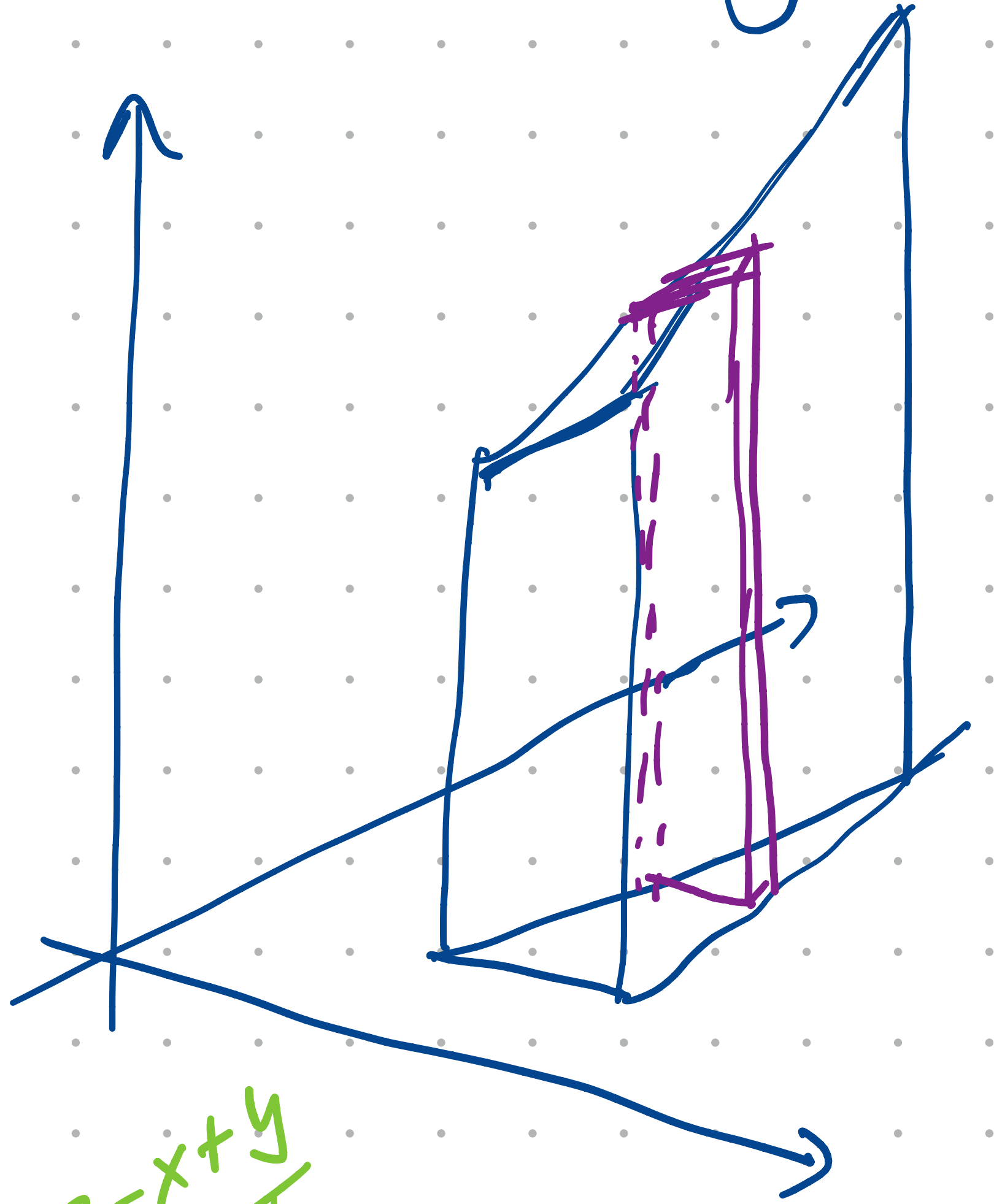
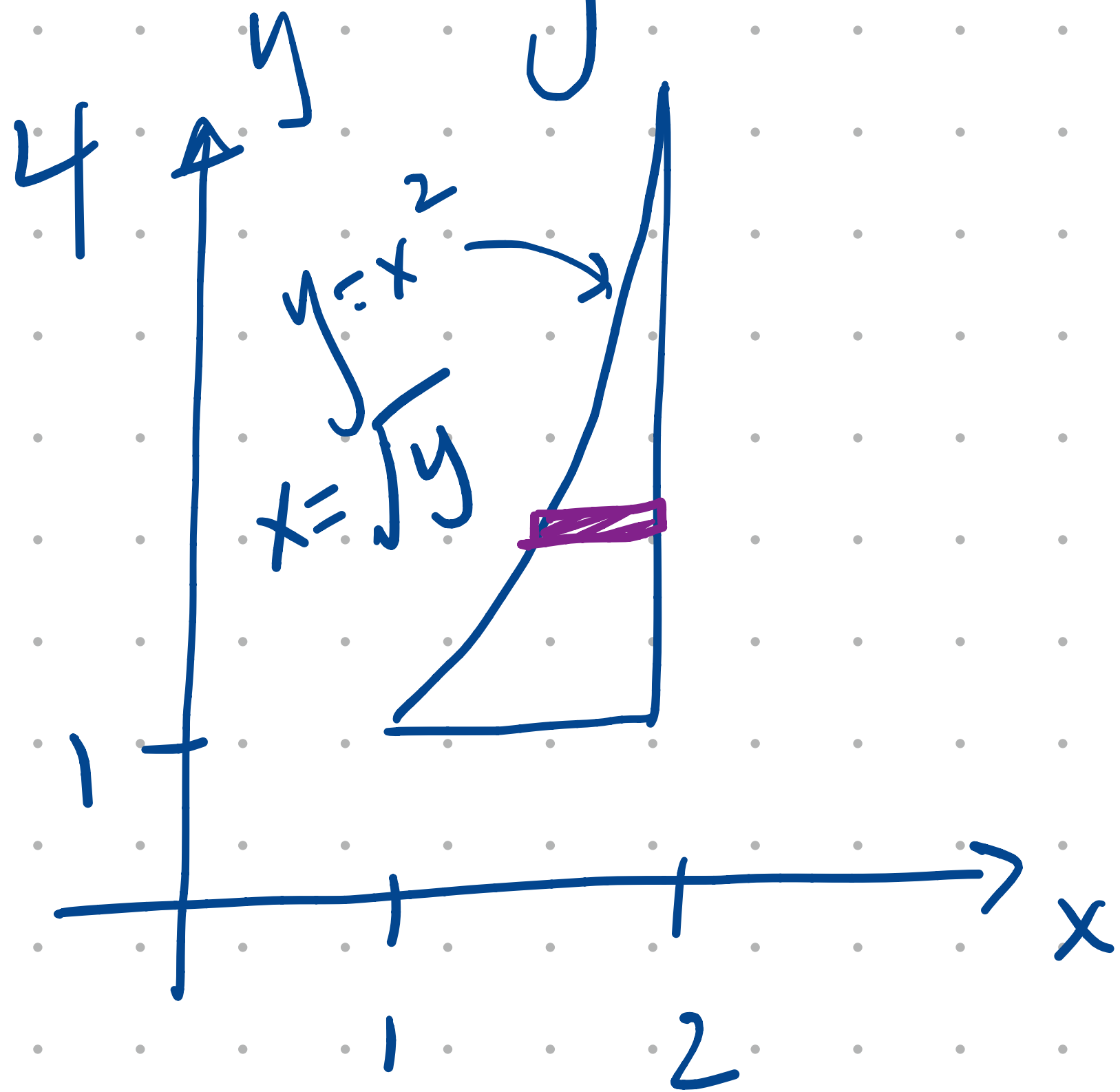
$$\int_1^2 \int_1^{x^2} (x+y) dy dx = \int_1^2 \left(xy + \frac{1}{2} y^2 \right) \Big|_{y=1}^{y=x^2} dx$$

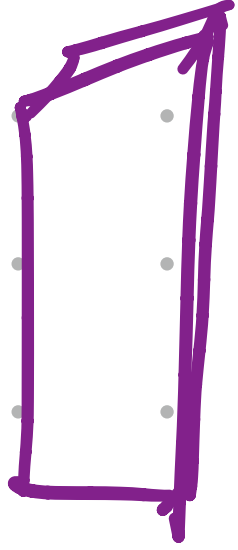
$$= \int_1^2 \left(x^3 + \frac{1}{2} x^4 \right) - \left(x + \frac{1}{2} \right) dx$$

$$= \left(\frac{1}{4} x^4 + \frac{1}{10} x^5 - \frac{1}{2} x^2 - \frac{1}{2} x \right) \Big|_{x=1}^2$$

$$= \frac{97}{20}$$

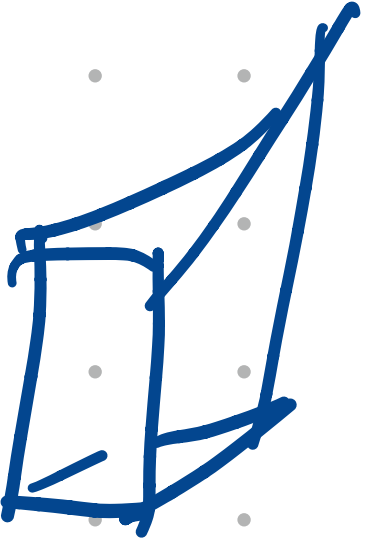
Just like last time, we can slice the region another way:



Volume of  = $\int \left(\int_{\sqrt{y}}^2 (x+y) dx \right) dy$

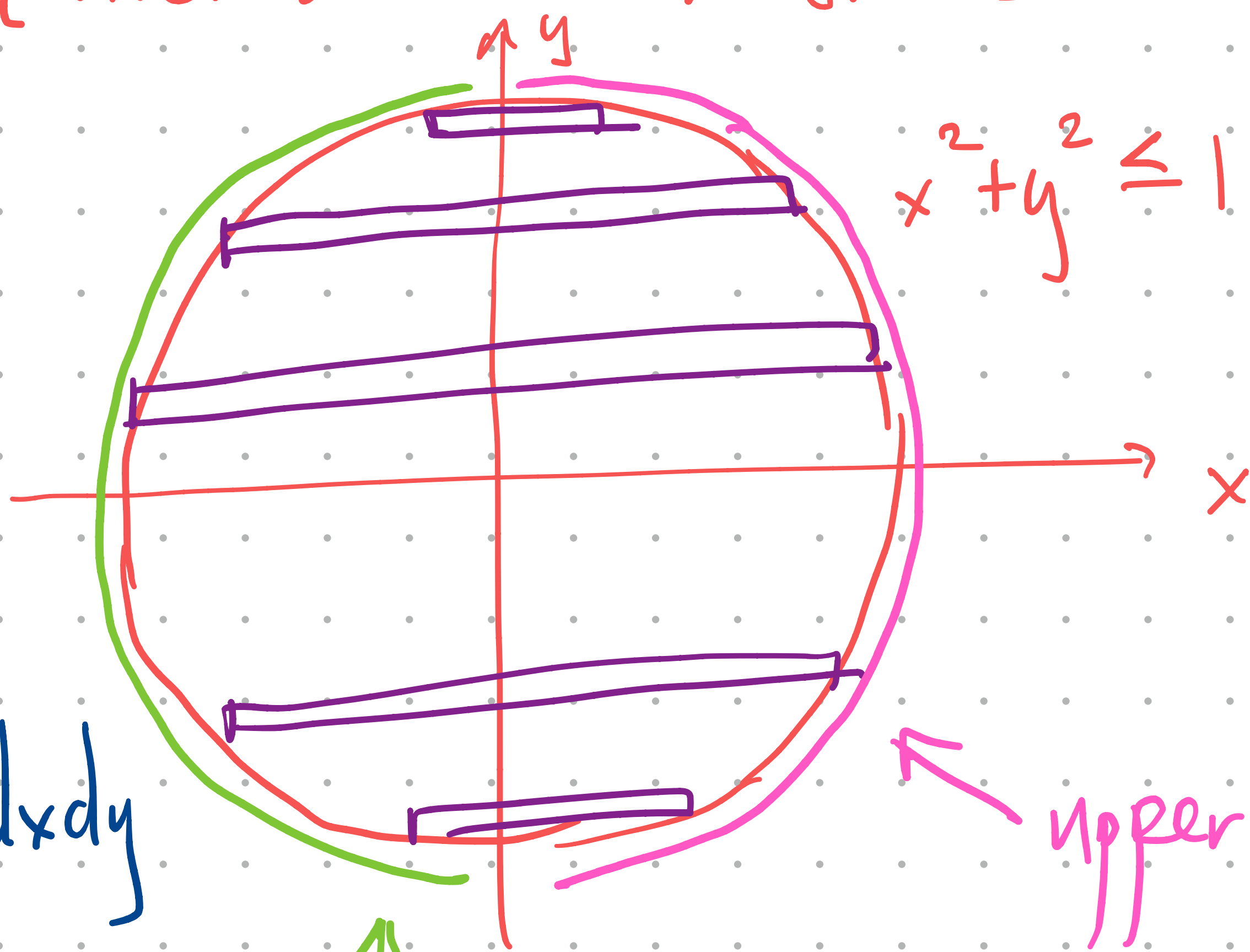
The equation shows the volume of a purple rectangular slice is equal to the integral of the area of a slice (a trapezoid) with respect to y. The trapezoid is shown in a green diagram with vertices at $(\sqrt{y}, 0)$, $(2, 0)$, $(2, z)$, and (\sqrt{y}, z) , where $z = x + y$.

$\int_{\sqrt{y}}^2 (x+y) dx$

Volume of  = $\int_1^4 \left(\int_{\sqrt{y}}^2 (x+y) dx \right) dy$

The final equation shows the total volume of the 3D object is the integral from y=1 to y=4 of the area of the slices.

When doing $dx dy$ integration order, x bounds are allowed to depend on y , but y bounds (outer bounds) must be constants.



$$x^2 + y^2 \leq 1$$

upper x-bound

$$x = \sqrt{1 - y^2}$$

lower x bound

$$x = -\sqrt{1 - y^2}$$

$$\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y) dx dy$$

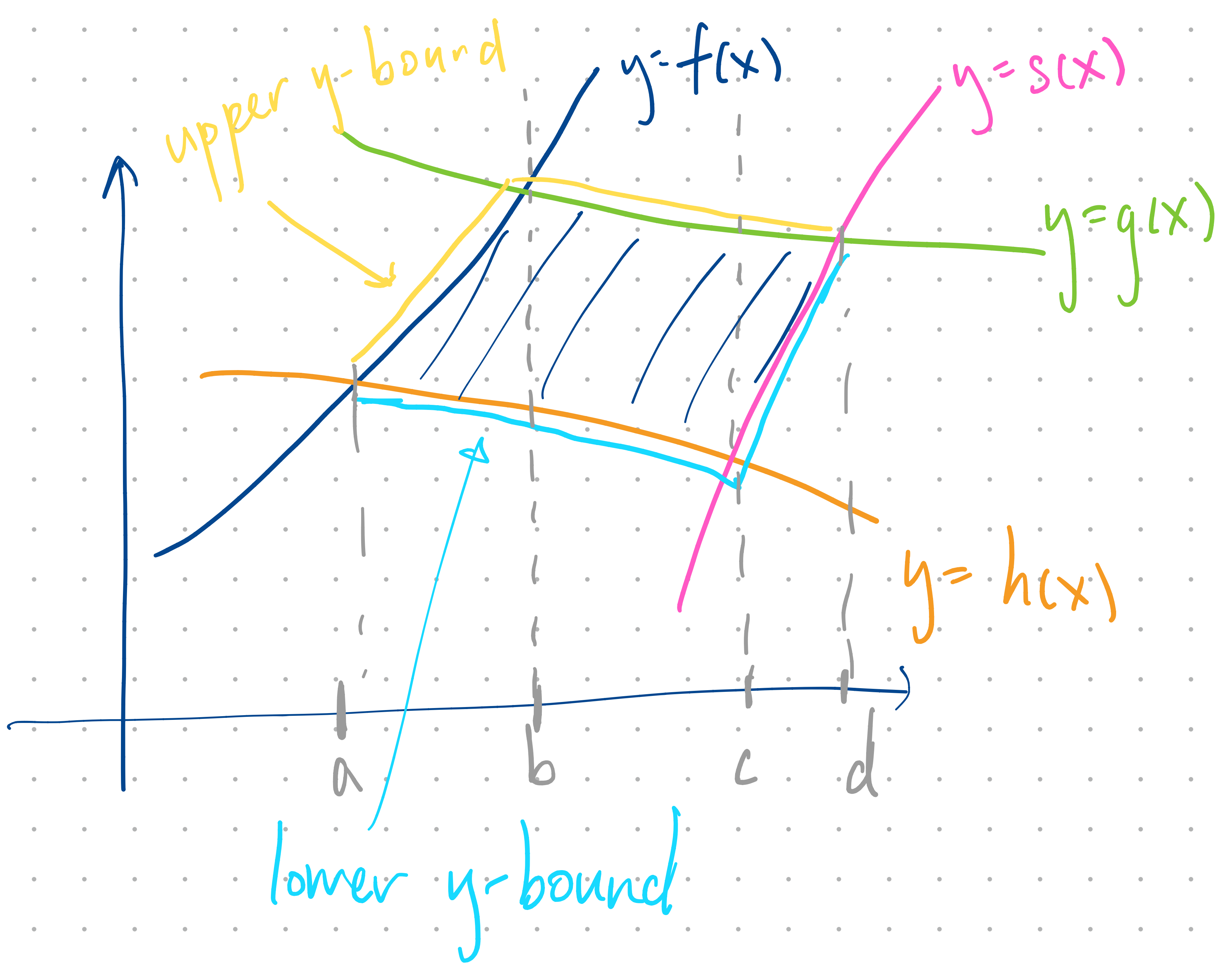
Compute:

$$\int_1^4 \int_{\sqrt{y}}^2 (x+y) dx dy = \int_1^4 \left(\frac{1}{2}x^2 + xy \right) \Big|_{x=\sqrt{y}}^2 dy$$

$$= \int_1^4 (2+2y) - \left(\frac{y}{2} + y^{3/2} \right) dy$$

$$= \left(2y + \frac{3}{4}y^2 - \frac{2}{5}y^{5/2} \right) \Big|_{y=1}^4$$

$$= \frac{97}{20}$$



If we wanted to integrate $u(x,y)$ over this region:

$$\int_a^b \int_{h(x)}^{f(x)} u(x,y) dy dx + \int_b^c \int_{h(x)}^{g(x)} u(x,y) dy dx + \int_c^d \int_{s(x)}^{g(x)} u(x,y) dy dx$$

🗨️ When poll is active, respond at PollEv.com/xianglongni346

📱 Text **XIANGLONGNI346** to **37607** once to join

Suppose we want to integrate $f(x, y)$ over the parallelogram bounded by the lines $y = x, y = 2x, y = x + 1, y = 2x - 1$. (Draw a picture.) If we use the integration order $dydx$, how many integrals do we have to write? (Try to actually write them.)

1

2

3

More than 3

Total Results: 9

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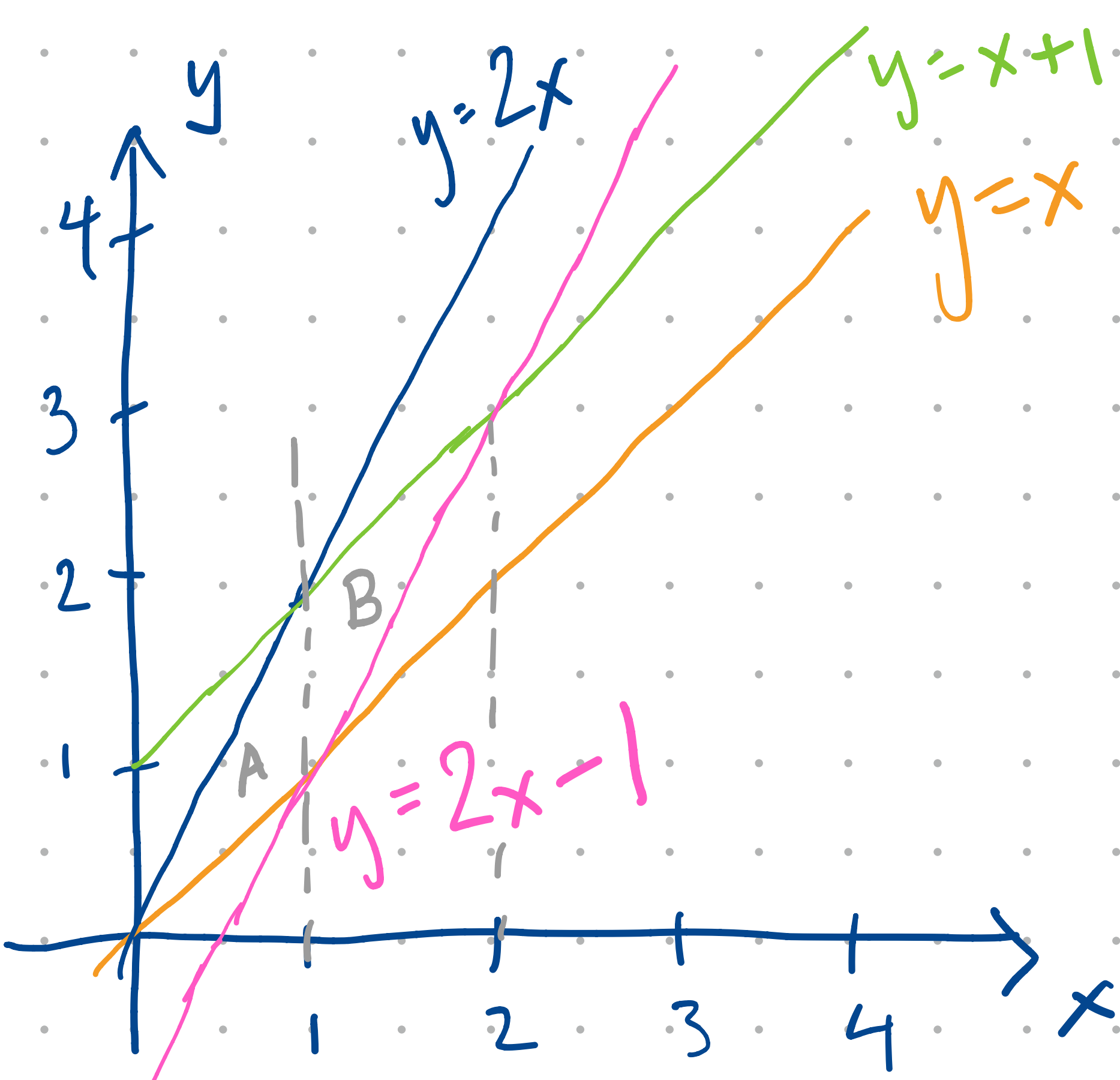
1

2

3

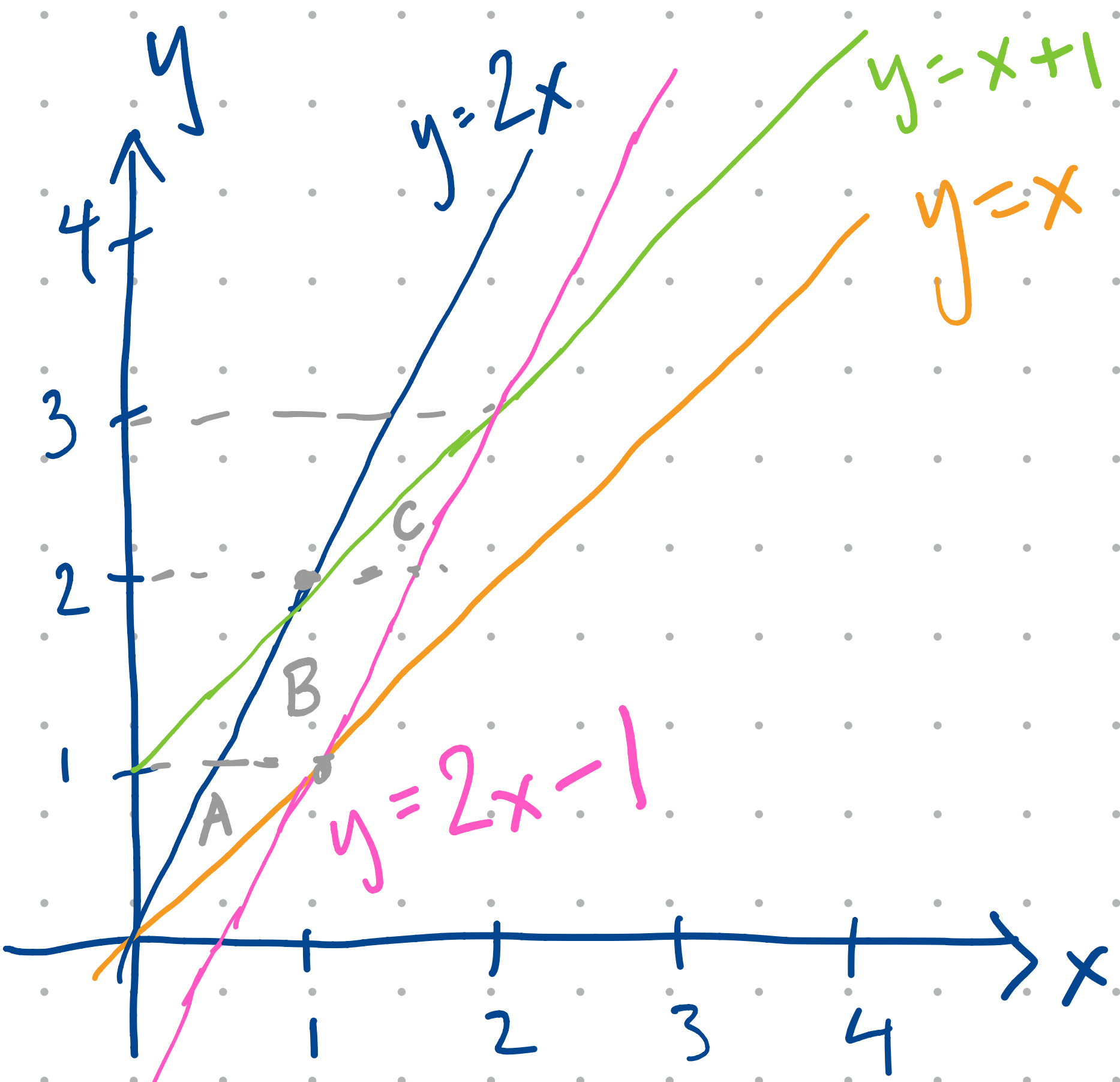
More than 3

Total Results: 8



$$\int_0^1 \int_x^{2x} f(x,y) dy dx \quad \textcircled{A}$$

$$+ \int_1^2 \int_{2x-1}^{x+1} f(x,y) dy dx \quad \textcircled{B}$$



$$\int_0^1 \int_{y/2}^y f(x,y) dx dy \quad \textcircled{A}$$

$$+ \int_1^2 \int_{y/2}^{(y+1)/2} f(x,y) dx dy \quad \textcircled{B}$$

$$+ \int_2^3 \int_{y-1}^{(y+1)/2} f(x,y) dx dy \quad \textcircled{C}$$